M3 - June 2002

1- $T = \frac{2\pi}{w}$

$w = \frac{2\pi}{2} = \pi$

$x = a\cos wt$

$x = 0.25\cos(\pi t)$

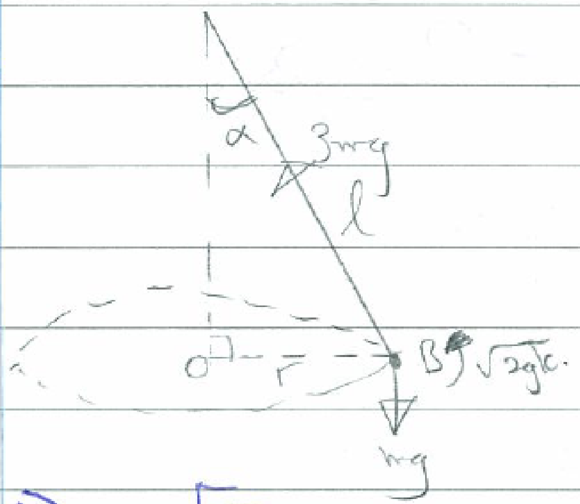
$-0.125 = 0.25\cos(\pi t)$

$-0.5 = \cos \pi t$

$\frac{2\pi}{3} = \pi t$

$t = \frac{2}{3}$

2-



a) $\uparrow$ $mg = 3mg\cos\alpha$

$\cos\alpha = \frac{1}{3}$

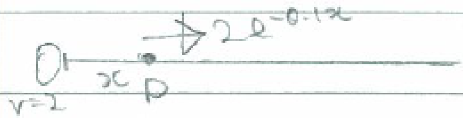
$\alpha = 70.5°$ (1dp)

b) $\leftarrow$ [F = ma]

$3mg\sin\alpha = m2gkr$

$3\sin\alpha = 2kl\sin\alpha$

$l = \frac{3}{2k}$

3_ 

a) $\rightarrow \boxed{F = ma}$

$2e^{-0.1x} = 2.5v\frac{dv}{dx}$

$2\int_0^x e^{-0.1x}\,dx = 2.5\int_2^v v\,dv$

$\frac{-2}{0.1}\left[e^{-0.1x}\right]_0^x = 1.25\left[v^2\right]_2^v$

$-20e^{-0.1x} + 20 = 1.25v^2 - 5$

$v^2 = 20 - 16e^{-0.1x}$

b) $4^2 = 20 - 16e^{-0.1x}$

$16e^{-0.1x} = 4$

$e^{-0.1x} = \frac{1}{4}$

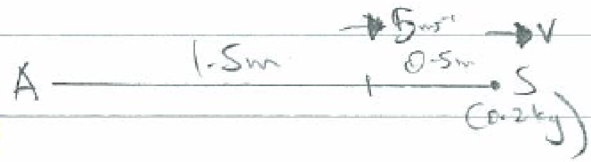
$-0.1x = \ln\frac{1}{4}$

$x = 10\ln 4$

c) As $x \rightarrow \infty$, $16e^{-0.1x} \rightarrow 0$

$\therefore v^2 \rightarrow 20$ and v cannot exceed $\sqrt{20}$ ms^{-1}

4. a)



$$\frac{1}{2}\times 0.2\times 5^2 = \frac{1}{2}\times 0.2\times v^2 + \frac{20\times 0.5^2}{3}$$

$$2.5 = 0.1v^2 + \frac{5}{3}$$

$$v^2 = \frac{50}{6}$$

$$v = 2.89\ ms^{-1}\ (3sf)$$

b) $$\frac{1}{2}(0.2)(5^2) = \frac{1}{2}(0.2)(1.5^2) + \frac{20x^2}{3}$$

$$2.5 = 0.225 + \frac{20}{3}x^2$$

$$6.825 = 20x^2$$

$$x^2 = 0.34125$$

$$x = 0.584m$$

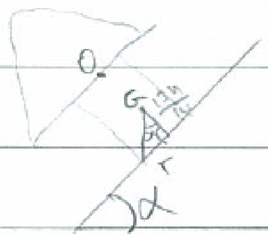
$$T = \frac{\lambda x}{a} = \frac{20 \times 0.584}{1.5} = 7.79\,\text{N} \ (3sf)$$

5. a) $$\frac{1}{3} \times \pi \times (2r)^2 \times h \times \frac{h}{4} + \pi r^2 h \times \frac{h}{2} = \left[\frac{1}{3}\pi (2r)^2 h + \pi r^2 h\right] OG$$

$$\frac{h}{2} - \frac{h}{3} = \frac{7}{3} OG$$

$$\frac{h}{6} = \frac{7}{3} OG$$

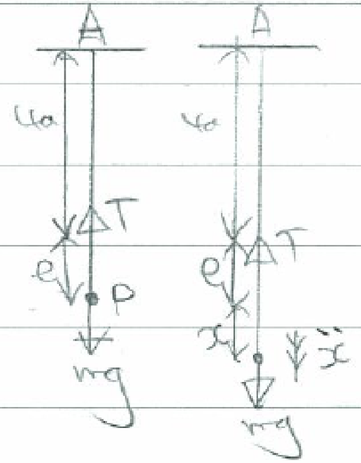
$$OG = \frac{3h}{6 \cdot 7} = \frac{1}{14} h$$

b) 

$$\tan\alpha = \frac{r}{\frac{13h}{14}} = r \times \frac{14}{13h} = \frac{7}{26}$$

$$r = \frac{7 \times 13h}{14 \cdot 26} = \frac{1}{4} h$$

6.



a) $T = mg$ $\qquad T = \frac{\lambda e}{a}$

$$\cancel{m}g = \frac{8\cancel{m}ge}{4a}$$

$$1 = \frac{8e}{4a}$$

$AO = 4a + \frac{1}{2}a = \frac{9}{2}a$ $\qquad e = \frac{a}{2}$

b) $T = \frac{\lambda x}{a} = \frac{8mg(e+x)}{4a} = \frac{2mge}{a} + \frac{2mgx}{a}$

$\downarrow \boxed{F = ma}$

$mg - T = m\ddot{x}$

$$\cancel{m}g - \frac{2\cancel{m}ge}{a} - \frac{2\cancel{m}gx}{a} = \cancel{m}\ddot{x}$$

$$g - \frac{2ga}{\;} - \frac{2gx}{\;} = \ddot{x}$$

$$\ddot{x} = -\frac{2g}{a}x$$

$\therefore$ SHM with $\omega^2 = \frac{2g}{a}$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2g}{a}}} = \frac{2\pi\sqrt{a}}{\sqrt{2g}} = \frac{2\pi\sqrt{2a}}{2\sqrt{g}} = \pi\sqrt{\frac{2a}{g}}$$

c) $\frac{1}{2}\sqrt{ga} = r\omega$

$\frac{1}{2}\sqrt{ga} = d \cdot \sqrt{\frac{2g}{a}}$

$$d = \frac{\sqrt{ga}\cdot\sqrt{a}}{2\sqrt{2g}}$$

$$= \frac{a}{2\sqrt{2}}$$

$$= \frac{a\sqrt{2}}{4}$$

d) The particle moves up with SHM until it becomes slack, then moves freely under gravity, then it starts SHM again when it is taut again

7- a) $ME_A = ME_P$

$\frac{1}{2}\cancel{m}u^2 = \cancel{m}g(l - l\cos\alpha)$

$u^2 = 2gl - 2gl\cdot\frac{2}{3}$

$= \frac{2}{3}gl$

$u = \sqrt{\frac{2gl}{3}}$

b) $\frac{1}{2}\cancel{m}u^2 = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g(l - l\cos\theta)$

$u^2 = v^2 + 2gl - 2gl\cos\theta$

$\frac{2gl}{3} = v^2 + 2gl - 2gl\cos\theta$

$v^2 = 2gl\cos\theta - \frac{4}{3}gl$

$\nwarrow [F = ma]$

$$T - \cancel{m}g\cos\theta = \frac{\cancel{m}}{\cancel{\ell}}\left(2g\cancel{\ell}\cos\theta - \frac{4}{3}g\cancel{\ell}\right)$$

$$T = mg\cos\theta + 2mg\cos\theta - \frac{4}{3}mg$$

$$= 3mg\cos\theta - \frac{4}{3}mg = \frac{mg}{3}(9\cos\theta - 4)$$

c) When $\cos\theta = \frac{2}{3}$, $T = \frac{mg}{3}\left(9 \cdot \frac{2}{3} - 4\right) = \frac{2mg}{3}$

When $\cos\theta = 1$, $T = \frac{mg}{3}(9-4) = \frac{5mg}{3}$

$$\therefore \frac{2mg}{3} \leqslant T \leqslant \frac{5mg}{3}$$